# Single-Hop Probing Asymptotics in Available Bandwidth Estimation: Sample-Path Analysis

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#### <u>Outline</u>

- Introduction
  - Constant -rate fluid cross-traffic model
  - Relationship to existing techniques
- General Bursty Cross-Traffic Model
- Experimental Verifications
- Implications to Existing Techniques
- Conclusion

Prior work

- Our work



• Measuring path avail-bw using probing streams:



 Basic question: the relationship between input, output, and the measurement goal: avail-bw

#### Single-Hop Fluid Model 1

- Assuming Constant-rate Fluid Cross-traffic — Constant Cross-traffic intensity  $\lambda$  in any time-interval — Constant Avail-bw  $A = C - \lambda$  in any time-interval
- Probing rate/gap of packet train
  - Probing gap: g

– Probing rate: 
$$r = s/g$$

Fluid models:

$$r_O = \begin{cases} C \frac{r_I}{r_I + \lambda} & r_I \ge C - \lambda \\ r_I & r_I \le C - \lambda \end{cases} \qquad g_O = \begin{cases} \frac{s + g_I \lambda}{C} & g_I \le \frac{s}{C - \lambda} \\ g_I & g_I \ge \frac{s}{C - \lambda} \end{cases}$$

 $P_n$ 

 $P_3$ 

(n-1)g

 $P_1$ 

 $P_2$ 

#### **Single-Hop Fluid Model 2**



#### How Existing Techniques Relate to Fluid Models



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#### **Extending to Bursty Cross-Traffic**

• For the gap model, we adapt it to

$$E[g_O] = \begin{cases} \frac{s+g_I\lambda}{C} & g_I \leq \frac{s}{C-\lambda} \\ g_I & g_I \geq \frac{s}{C-\lambda} \end{cases}$$

- $g_O$  now varies, we change it to the asymptotic average  $E[g_O] = \lim_{k \to \infty} \frac{1}{k} \sum_{i=1}^k g_O^{(i)}$
- Cross-traffic rate is no longer a constant,  $\lambda$  is interpreted as its long-term average.

$$\lambda = \lim_{t \to \infty} \frac{V(t)}{t}$$

#### **Real Asymptotic Model**

• With proof, we offer the following gap model in bursty cross-traffic:

$$E[g_O] = \begin{cases} \frac{s+g_I\lambda}{C} + \frac{E[\tilde{I}(t,t+(n-1)g_I)]}{n-1} & g_I \leq \frac{s}{C-\lambda} \\ g_I + \frac{E[R_n(t)]}{n-1} & g_I \geq \frac{s}{C-\lambda} \end{cases}$$

 The two additional terms are zero in fluid traffic, but are often POSITIVE in bursty cross-traffic.

# What is the term $E[R_n(t)]/(n-1)$

•  $R_n(t)$  is the <u>additional</u> queuing delay imposed on the last packet  $P_n$  by the first n-1 packets in the same probing train when the train arrives into the hop at time t. It is called <u>intrusion residual</u>.



• $E[R_n(t)]$  is the asymptotic time average of  $R_n(t)$ 

$$E[R_n(t)] = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} R_n(t) dt$$

#### What is the term

 $E[\tilde{I}(t,t+(n-1)g_I)]$ n-1

$[t, t + (n - 1)g_I)]$	The measurement interval of a packet train when it arrives to the hop at time $t$ .
$\tilde{I}(t,t+(n-1)g_I)$	The amount of hop idle time in that measurement interval after the hop is visited by the packet train at time $t$ .
$E[\tilde{I}(t,t+(n-1)g_I)]$	Asymptotic time average of the hop idle time within the measurement interval of a packet train.

# What is the term $\frac{E[\tilde{I}(t,t+(n-1)g_I)]}{n-1}$



#### **Probing Bias**

•The following two terms, called probing bias, are the difference between fluid model and real asymptotic model.

$$\beta(g_I, s, n) = \begin{cases} \frac{E[\tilde{I}(t, t + (n-1)g_I)]}{n-1} & g_I \leq \frac{s}{C-\lambda} \\ \frac{1}{n-1}E[R_n(t)] & g_I \geq \frac{s}{C-\lambda} \end{cases}$$

•The closed-form expression of probing bias is given in the paper.

#### Probing Bias VS. Input Gap g



When  $g_{I} > s/A$ , bias monotonically decreases and asymptotically converges to 0.

#### Gap Model in Bursty Cross-traffic



#### Rate Model in Bursty Cross-traffic



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#### **Impact of Packet-train Parameters**

- Larger packet size pushes the real model closer to the fluid model
  - Sampling interval increases, cross-traffic variance decreases, cross-traffic is more like fluid.
- Longer packet train also pushes the real model closer to the fluid model.
  - Non-intuitive, the paper offers an explanation using random walk theory.
- Fluid models are tight bounds for real models

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#### Period Testing 1

- The deviation phenomena were first observed in periodic cross-traffic such as CBR
- $E[g_O]$  can be easily computed , since it is equal to the time average of  $g_O(t)$  in one period:

$$E[g_O] = \frac{1}{T} \int_0^T g_O(t) dt$$

- —Where  $g_O(t)$  is the output gap of a packet train when it arrives at the hop at time t.
- Notice that  $g_O(t)$  is also a periodic function of time with the same period T as that of the cross-traffic.

### Period Testing 2

- Period Testing approximates the time average of  $g_O(t)$  in [0, T]
  - By sampling it at a set of equally spaced time instances and taking the average of those samples.
- The number of samples is chosen so that
  - Using more samples makes little difference
  - Results agree with fluid model when  $0 < g_I < s/C$

#### Packet-Pair Rate Curve in CBR

CBR cross-traffic with average intensity 2.5mb/s, Hop capacity *C*=10mb/s 8 s/E[<sub>*g*O] (mb/s)</sub> 6 As probe packet size increases, both the deviation range and 4 deviation amplitude shrink. Rate upper bound Probe size 1500B 2 Probe size 750B Probe size 250B Rate lower bound 0 2 6 10 12 14 8 4 0 Input Rate  $r_{\rm I}$  (mb/s)

#### Packet-train Rate Curve in CBR



#### **Trace-driven Testing**

- Allows examining the asymptotic model in different types of cross-traffic
- Use time average of  $g_O(t)$  in a finite time interval  $[0, \alpha]$  to approximate  $E[g_O]$
- α is chosen so that the cross-traffic intensity in [0,
  α] is close to its long term average
- $g_O(t)$  can be computed based on cross-traffic trace and hop capacity *C*, when  $t+(n-1)g_I < \alpha$

#### **Cross-traffic Traces**



#### Packet-Pair Rate Curves



#### Packet-Train Rate Curves



#### Probing bias VS. Cross-traffic Burstiness

- The results so far shows that:
  - As probing packet size or train length increases, probing bias vanishes.
  - The vanishing rate depends on cross-traffic burstiness. CBR>Poisson>Pareto on/off
  - Although Pareto on/off is more bursty than Poisson, at certain time interval, the traffic variance can be smaller than Poisson, causing less probing bias in its rate curve.
- More discussion is in the paper

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#### Implication to existing techniques 1

• TOPP use a transformed rate curve which is piece-wise linear in fluid cross-traffic

$$\frac{r_I}{r_O} = \begin{cases} 1 & r_I \le C - \lambda \\ \frac{r_I + \lambda}{C} & r_I \ge C - \lambda \end{cases}$$

 Real asymptotic curves are not the same as the fluid models. This can cause significant under estimation of avail-bw even in a single-hop path

#### Implication to existing techniques 2



#### Implication to existing techniques 3

- Searching for the turning point (PTR) as available bandwidth causes negative bias
  - However, this bias can be mitigated to negligible level using long packet train.
- Sampling cross-traffic (Spruce) with r<sub>l</sub>≥C is unbiased in single-hop path
  - At this input rate, the real model agrees with fluid model.

#### **Conclusions**

- We developed an understanding of single-hop bandwidth estimation in busty cross-traffic that extends prior fluid models
- Cross-traffic burstiness implies bandwidth underestimation to several existing techniques. The underestimation can be mitigated using long train and large packet size
- Future work is to extend our understanding to multi-hop bandwidth estimation

